Energy growth for the solutions of the nonlinear wave equation with time periodic potential

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Abstract. It is known that for some time periodic potentials $q(t,x) \geq 0$ having compact support with respect to x some solutions of the Cauchy problem for the wave equation $\partial_t^2 u - \Delta_x u + q(t,x)u = 0$ have exponentially increasing energy as $t \to \infty$. This phenomenon is called parametric resonance. We show that if one adds a nonlinear defocusing interaction $|u|^r u, 2 \leq r < 4$, then the solution of the nonlinear wave equation exists for all $t \in \mathbb{R}$ and its energy is polynomially bounded as $t \to \infty$ for every choice of the periodic potential $q(t,x)$. Moreover, we prove that the zero solution of the nonlinear wave equation is instable if the corresponding linear equation has the property mentioned above. This is a joint work with N. Tzvetkov.