

The semi-classical Bogoliubov-de Gennes Hamiltonian with PT-symmetry: generalized Bohr-Sommerfeld quantization rules

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Bogoliubov-de Gennes (BdG) Hamiltonian describes the dynamics of a pair of quasi-particles in SNS junctions. A narrow metallic lead, with few transverse channels, connecting two superconducting contacts, is identified with a 1-D structure $|x| \leq L - \ell/2$, $\ell \ll L$ measuring the “penetration length”. Interaction with the superconductor bulk is modeled through the complex order parameter, $\Delta_0 e^{i\phi(x)/2}$ for $|x| \geq L + \ell/2$, $\phi(x) = \text{sgn}(x)\phi$; because of the finite range of the junction (comparable to ℓ) we may assume that this interaction continues to a smooth function $x \mapsto \Delta(x)e^{i\phi(x)}$, which vanishes rapidly inside $[-L, L]$. BdG Hamiltonian then takes the form

$$\mathcal{P}(x, \xi) = \begin{pmatrix} \xi^2 - \mu(x) & \Delta(x)e^{i\phi(x)/2} \\ \Delta(x)e^{-i\phi(x)/2} & -\xi^2 + \mu(x) \end{pmatrix} \quad (1)$$

We consider dynamics of the quasi-particle described semi-classically by the operator $\mathcal{P}(x, hD_x)$ on $L^2(\mathbb{R}) \otimes \mathbb{C}^2$. To simplify the model we have continued $\mu(x)$ to a constant μ_0 for $|x| \geq L + \ell/2$, which makes sense if ℓ is sufficiently large with respect to the typical wave-length h . We assume potentials $x \mapsto \mu(x)$ and $x \mapsto \Delta(x)$ to be smooth and even functions on the real line, so that $\mathcal{P}(x, hD_x)$ enjoys time-reversal and PT symmetries. Electrons (e^-) and holes (e^+) with energy $E < \inf_{\mathbb{R}} \mu(x)$, $E < \Delta_0$ form so-called Andreev states sensitive to the variation of phase parameter ϕ between the superconducting banks. We are interested in resonances for the scattering processes $e^- \rightarrow e^+$ and $e^+ \rightarrow e^-$ where the wave functions of e^\pm are purely outgoing at infinity (within the approximation above). We compute the first order asymptotics of the real parts of resonances, as solutions of generalized Bohr-Sommerfeld quantization rules, by constructing “relative monodromy operators” in the “classically allowed region” defined by $\Delta(x) \leq E$. These operators belong to $U(1,1)$ the unitary group associated to the “flux norm” (in fact, a Lorenzian metric) generalizing a concept introduced by Helffer and Sjöstrand.